AN EXTENSION OF MASSEY'S DISTRIBUTION OF THE MAXIMUM DEVIATION BETWEEN TWO SAMPLE CUMULATIVE STEP FUNCTIONS

by

CHIA KUEI TSAO

TECHNICAL REPORT NO. 7

July 15, 1953

PREPARED UNDER CONTRACT Nonr-451(00)
(NR-042-017)

FOR

OFFICE OF NAVAL RESEARCH

DEPARTMENT OF MATHEMATICS
WAYNE UNIVERSITY
DETROIT, MICHIGAN

AN EXTENSION OF MASSEY'S DISTRIBUTION OF THE MAXIMUM DEVIATION BETWEEN TWO SAMPLE CUMULATIVE STEP FUNCTIONS

By Chia Kuei Tsao
Wayne University

1. Summary and Introduction

Let $x_1 < x_2 < \ldots < x_n$ and $y_1 < y_2 < \ldots < y_m$ be the ordered results of two random samples from populations having continuous cumulative distribution functions $F(x)$ and $G(x)$ respectively. Let $S_n(x) = k/n$ where k is the number of observations of X which are less than or equal to x and $S_m'(x) = j/m$ where j is the number of observations of Y which are less than or equal to x.

The statistics

$$d_r = \max_{x \leq x_r} \left| S_n(x) - S_m'(x) \right|$$

and

$$d_r' = \max_{\substack{x \leq \max(x_r, y_r) \\ r \leq \min(m,n)}} \left| S_n(x) - S_m'(x) \right|$$

can be used to test the hypothesis $F(x) = G(x)$. For example, using d_r we would reject the hypothesis if the observed d_r, i.e., the maximum absolute deviation between the two step functions below the rth observation of a given sample, is significantly large.

2. Distribution of d_r

Denote by m_1 the number of observed values of Y which are less than x_1, by m_2 the number of values of Y which are between x_1 and x_2, ..., by m_r the number of values of Y which are between x_{r-1} and x_r and by M the number of values of Y which are greater than x_r. If the hypothesis $F(x) \equiv G(x)$ is true, the probability of the occurrence of a set of $m_1, m_2, \ldots, m_r, M$ can be shown to be

$$\Pr(m_1, \ldots, m_r, M) = \frac{\binom{M+n-r}{M}}{\binom{m+n}{m}}$$

which depends only on M, i.e., for any given M the probability of the occurrence of any set of $m_1, m_2, \ldots, m_r$ is always $\binom{M+n-r}{M} \Big/ \binom{m+n}{m}$.
Thus, for any given M, the probability that $d_r \leq a$ can be found by counting the number of sets of $m_1, m_2, \ldots, m_r$ which give values of $d_r \leq a$.
Denote this number of sets by $K_{r,M}(a)$, then

$$\Pr(d_r \leq a) = \sum_{M=0}^{m} K_{r,M}(a) \frac{\binom{M+n-r}{M}}{\binom{m+n}{m}} .$$

The method of counting $K_{r,M}(a)$ is essentially the same as in [5]. As an illustration, suppose m=n, then $S_m(x)$ and $S'_m(x)$ can only differ by multiples of $\frac{1}{m}$. For any integer c and any given M, we count the number of sets of $m_1, m_2, \ldots, m_r$ such that $d_r \leq \frac{c}{m}$.

Denote by $V_{ij}(c)$, $i = 1,2,\ldots,r$, $j = 1,2,\ldots,2c$, the number of sets of possible $m_1, m_2, \ldots, m_i$ such that $d_i \leq \frac{c}{m}$. Then it is evident

that these $V_{ij}(c)$ satisfy the following difference equations

$$V_{ik}(c) = \sum_{j=1}^{k+1} V_{i-1,j}(c) \qquad \begin{array}{l} i = 1,2,\ldots r \\ k = 1,2,\ldots 2c \end{array}$$

where

$$V_{0k}(c) = 1 \qquad k \geq c,\ k = 1,2,\ldots,2c$$

$$V_{0j}(c) = 0 \qquad \text{otherwise.}$$

Hence,

$$K_{r,M}\left(\frac{c}{m}\right) = V_{r,m-M-r+c+1}(c).$$

3. Distribution of d'_r

If $r \leq n\, d_r$, then a test based on the number of observations of one sample which are less than or equal to the rth observation of another sample becomes a one-sided test. To avoid this, we may use the statistic d'_r. In this case

$$\Pr(d'_r \leq a) = \Pr(d_r \leq a,\ x_r > y_r) + \Pr(d_r \leq a,\ x_r < y_r)$$

$$= \sum_{M=0}^{m-r} K_{r,M}(a) \frac{\binom{M+n-r}{M}}{\binom{m+n}{m}}$$

$$+ \sum_{N=0}^{n-r} K_{r,N}(a) \frac{\binom{N+m-r}{N}}{\binom{m+n}{m}}$$

where $n_1, n_2, \ldots, n_r, N$ have the same meaning as $m_1, m_2, \ldots, m_r, M$.

If $m = n$, then

$$Pr(d'_r \le a) = 2 \sum_{M=0}^{m-r} K_{r,M}(a) \frac{\binom{M+m-r}{M}}{\binom{2m}{m}} .$$

We note that:

(a) If $r = m = n$, then both distributions of d_m and d'_m reduce to Massey's distribution [5].

(b) If $r = 1$, then d_r reduces to a special case of Gumbel and Schelling's exceedances problems [2].

Table I and II give the probabilities of d_r and d'_r respectively for $m = n$.

4. Applications

The statistics d_r and d'_r are useful for situations where the sample sizes are known, but the information beyond a certain ordered observation, say x_r, is unavailable. In life testing, one often wishes, by drawing two samples, to detect whether the mean life in one population is larger than that of another. If the observations become available in order of magnitude, then we can stop the experiment whenever at least r observations of each sample have occurred and reach a decision by the use of d'_r. Evidently, by doing so, it would be possible, in many cases, to reduce both the average time needed and/or the average number of items destroyed.

As an illustration, we give a numerical example as follows: Suppose fuses are produced by two different methods. One is interested in detecting whether the mean current needed to blow the fuses produced by the

first method is different from that produced by the second. This can be considered as testing whether two populations are the same. To this end, one then may put, say, 40 fuses produced by the first method and another 40 by the second on a test. Suppose one arranges the test in such a way that every fuse in the 2 samples is subjected to the same current so that the weakest blows first, then the second, etc. Let us choose in advance that $r = 6$ and $\alpha = .05$. Let $x_1 < x_2 < \dots$ denote the ordered observed current needed to blow the fuses in the first sample and $y_1 < y_2 < \dots$ those in the second. Suppose that the actual combined outcomes are as follows: $x_1 x_2 x_3 x_4 y_1 x_5 x_6 x_7 y_2 x_8 x_9 y_3 x_{10} x_{11} x_{12} \dots$ then the experiment may be terminated when the observation x_{12} has occurred and reject the null hypothesis using the statistic d_6', since for $m = n = 40$, $Pr(d_6' \geq 9) = .04\overset{9}{5}1$ from Table II. In this particular experiment, only 20% of the fuses are destroyed in reaching a decision.

It, perhaps, should be remarked that if we define

$$D_r = \underset{x \geq x_{n-r+1}}{\mathrm{Max}} \left| S_n(x) - S_m'(x) \right|$$

$$D_r' = \underset{\substack{x \geq \overset{min}{} (x_{n-r+1},\, y_{m-r+1}) \\ r \leq \mathrm{Min}\,(m,n)}}{\mathrm{Max}} \left| S_n(x) - S_m'(x) \right|$$

then the distribution of D_r, and D_r' are identical with those of d_r and d_r'. Thus, in a test, if the information below a certain ordered observation is unavailable, or if the observations become available in this order:

$x_n, x_{n-1}, \ldots x_1$ and $y_m, y_{m-1}, \ldots y_1$, then D_r or D_r' would be the appropriate statistic to use.

In conclusion, I would like to thank Mrs. Dorothy Wolfe who carried out the computations of Tables I and II.

References

1. W. Feller, "On the Kolmogorov-Smirnov Limit Theorems for Empirical Distributions", Annals of Math. Stat., Vol. 19, (1948), pp. 177-189.

2. E. J. Gumbel and H. von Schelling, "The Distribution of the number of exceedances", Annals of Math. Stat., Vol. 21 (1950), pp. 247-262.

3. F. J. Massey, Jr., "A note on the estimation of a distribution function by confidence limits", Annals of Math. Stat., Vol. 21, (1950), pp. 116-119.

4. F. J. Massey, Jr., "A note on the power of a non-parametric test", Annals of Math. Stat., Vol. 21, (1950), pp. 440-443.

5. F. J. Massey, Jr., "The distribution of the maximum deviation between two sample cumulative step functions", Annals of Math. Stat., Vol. 22, (1951), pp. 125-128.

6. N. Smirnov, "Tables for estimating the goodness of fit of empirical distributions", Annals of Math. Stat., Vol. 19, (1948), pp. 279-281.

7. S. S. Wilks, "Statistical prediction with special reference to the problem of tolerance limits", Annals of Math. Stat., Vol. 13, (1942), pp. 400-409.

Table I

Probability of $d_r \leq c/m$

m	r \ c	1	2	3	4	5	6	7	8	9	10	11	12
3	2	.50000	.95000	1.00000									
4	2	.45714	.81429	.98571	1.00000								
	3	.28571	.81429	.98571	1.00000								
5	2	.43651	.85714	.96032	.99603	1.00000							
	3	.25397	.73810	.96032	.99603	1.00000							
	4	.15873	.67857	.93651	.99603	1.00000							
6	2	.42424	.84091	.94372	.98701	.99892	1.00000						
	3	.23810	.70130	.92857	.98701	.99892	1.00000						
	4	.13853	.60390	.89827	.98701	.99892	1.00000						
	5	.08658	.55519	.87229	.97944	.99892	1.00000						
7	2	.41608	.83042	.93240	.97931	.99592	.99971	1.00000					
	3	.22844	.67920	.90793	.97348	.99592	.99971	1.00000					
	4	.12821	.56643	.85897	.97348	.99592	.99971	1.00000					
	5	.07459	47776 ~~.38548~~	.82634	.96300	.99592	.99971	1.00000					
	6	.04662	.44843	.80186	.95280	.99359	.99971	1.00000					

Table I (continued)

m	r \ c	1	2	3	4	5	6	7	8	9	10	11	12
8	2	.41026	.82308	.92424	.97319	.99277	.99876	.99992	1.00000				
	3	.22191	.66434	.89378	.96232	.99068	.99876	.99992	1.00000				
	4	.12183	.54336	.83294	.95649	.99068	.99876	.99992	1.00000				
	5	.06838	.45315	.78291	.94367	.99068	.99876	.99992	1.00000				
	6	.03978	.39021	.75245	.92968	.98718	.99876	.99992	1.00000				
	7	.02486	.35874	.73007	.91880	.98345	.99806	.99992	1.00000				
9	2	.40588	.81765	.91810	.96833	.98992	.99757	.99963	.99998	1.00000			
	3	.21719	.65362	.88355	.95352	.98548	.99685	.99963	.99998	1.00000			
	4	.11748	.52756	.81473	.94272	.98322	.99685	.99963	.99998	1.00000			
	5	.06450	.43149	.75376	.92236	.98322	.99685	.99963	.99998	1.00000			
	6	.03620	.35985	.70769	.90539	.97869	.99685	.99963	.99998	1.00000			
	7	.02106	.30987	.68005	.89058	.97314	.99572	.99963	.99998	1.00000			
	8	.01316	.28488	.65981	.87982	.96870	.99443	.99942	.99998	1.00000			

Table I (continued)

m	r	1	2	3	4	5	6	7	8	9	10	11	12
10	2	.40248	.81347	.91331	.96440	.98744	.99637	.99921	.99989	.99999	1.00000		
	3	.21362	.64551	.87580	.94653	.98086	.99466	.99897	.99989	.99999	1.00000		
	4	.11431	.51602	.80128	.93192	.97610	.99383	.99897	.99989	.99999	1.00000		
	5	.06183	.41650	.73309	.90525	.97378	.99383	.99897	.99989	.99999	1.00000		
	6	.03395	.34065	.67739	.88049	.96836	.99383	.99897	.99989	.99999	1.00000		
	7	.01952	.28409	.63587	.86262	.96121	.99228	.99897	.99989	.99999	1.00000		
	8	.01108	.24464	.61101	.84804	.95164	.99020	.99861	.99989	.99999	1.00000		
	9	.00693	.22491	.59283	.83759	.94987	.98849	.99818	.99983	.99999	1.00000		
15	2	.39272	.80172	.89962	.95259	.97920	.99159	.99690	.99898	.99970	.99993	.99999	1.00000
	3	.20383	.62328	.85472	.92635	.96571	.98547	.99447	.99815	.99947	.99987	.99998	1.00000
	4	.10611	.48591	.76593	.90199	.95254	.97933	.99203	.99735	.99926	.99994	.99997	1.00000
	5	.05544	.38006	.68171	.85897	.94046	.97372	.98990	.99671	.99913	.99982	.99997	1.00000
	6	.02909	.29843	.60791	.81327	.92146	.96902	.98828	.99632	.99908	.99982	.99997	1.00000
	7	.01534	.23544	.54425	.77033	.90032	.96232	.98728	.99617	.99908	.99982	.99997	1.00000
	8	.00814	.18683	.48977	.73200	.88009	.95482	.98578	.99617	.99908	.99982	.99997	1.00000
	9	.00436	.14935	.44362	.69904	.86243	.94774	.98378	.99582	.99908	.99982	.99997	1.00000
	10	.00236	.12055	.40521	.67192	.84825	.94159	.98135	.99515	.99898	.99982	.99997	1.00000

Table I (continued)

m	r \ c	1	2	3	4	5	6	7	8	9	10	11	12
20	2	.38808	.79626	.89313	.94674	.97476	.98868	.99519	.99808	.99928	.99975	.99992	.99998
	3	.19939	.61316	.84525	.91681	.95784	.97989	.99100	.99624	.99854	.99948	.99983	.99995
	4	.10260	.47286	.75053	.88858	.94070	.97047	.98627	.99410	.99767	.99916	.99973	.99992
	5	.05289	.36526	.66048	.83900	.92417	.96103	.98143	.99186	.99674	.99883	.99963	.99990
	6	.02733	.28268	.58122	.78559	.89854	.95146	.97672	.98969	.99580	.99853	.99954	.99988
	7	.01414	.21923	.51231	.73418	.86907	.93937	.97236	.98773	.99512	.99829	.99948	.99987
	8	.00734	.17043	.45256	.68651	.83906	.92458	.96661	.98607	.99453	.99813	.99945	.99986
	9	.00382	.13287	.40080	.64319	.81020	.90909	.96046	.98417	.99414	.99804	.99944	.99986
	10	.00200	.10393	.35605	.60405	.78336	.89401	.95397	.98198	.99011	.99802	.99944	.99986
30	2	.38359	.79103	.88687	.94096	.97024	.98549	.99316	.99688	.99863	.99942	.99976	.99991
	3	.19520	.60360	.83640	.90768	.94998	.97393	.98693	.99370	.99708	.99870	.99944	.99977
	4	.09940	.46086	.73634	.87616	.92922	.96122	.97963	.98974	.99504	.99771	.99899	.99957
	5	.05065	.35211	.64144	.82088	.90888	.94809	.97170	.98525	.99265	.99651	.99842	.99932
	6	.02583	.26922	.55808	.76114	.87750	.93498	.96350	.98045	.99001	.99515	.99776	.99902
	7	.01318	.20600	.48576	.70332	.84110	.91690	.95526	.97550	.98724	.99369	.99705	.99870
	8	.00673	.15775	.42312	.64937	.80340	.89530	.94486	.97055	.98442	.99220	.99631	.99836
	9	.00344	.12092	.34041	.59969	.76628	.87193	.93244	.96472	.98164	.99072	.99559	.99803
	10	.00176	.09278	.32189	.55418	.73062	.84800	.91367	.94868	.97355	.98930	.99490	.99773

Table I (continued)

c 1		2	3	4	5	6	7	8	9	10	11	12
2	.38239	.78851	.88382	.93811	.96793	.98361	.99203	.99617	.99821	.99918	.99963	.99984
3	.19319	.59901	.83218	.90327	.94607	.97086	.98472	.99221	.99614	.99814	.99913	.99960
4	.09790	.45521	.72965	.87031	.92365	.95655	.97606	.98722	.99339	.99668	.99838	.99924
5	.04962	.34605	.72363	.81244	.90164	.94169	.96659	.98148	.99007	.99484	.99741	.99874
6	.02517	.25940	.54759	.74992	.86770	.92678	.95669	.97524	.98633	.99270	.99623	.99812
7	.01277	.20022	.47403	.68948	.82829	.90623	.94661	.96869	.98228	.99032	.99489	.99740
8	.00648	.15239	.41049	.63311	.78741	.88163	.93389	.96198	.97803	.98776	.99342	.99660
9	.00329	.11604	.35572	.58144	.74735	.85523	.91899	.95439	.97402	.98543	.99221	.99608
10	.00167	.08840	.30821	.53369	.70817	.82735	.90161	.94461	.96875	.98234	.99025	.99482

Table II

Probability of $d'_r \leq c/m$

m	r \ c	1	2	3	4	5	6	7	8	9	10	11	12
3	2	.40000	.90000	1.00000									
4	2	.34286	.77143	.97143	1.00000								
	3	.22857	.77143	.97143	1.00000								
5	2	.31746	.71429	.92063	.99206	1.00000							
	3	.19048	.64286	.92063	.99206	1.00000							
	4	.12698	.64286	.92063	.99206	1.00000							
6	2	.30303	.68182	.88745	.97403	.99784	1.00000						
	3	.17316	.58442	.85714	.97403	.99784	1.00000						
	4	.10390	.52597	.85714	.97403	.99784	1.00000						
	5	.06926	.52597	.85714	.97403	.99784	1.00000						
7	2	.29371	.66084	.86480	.95862	.99184	.99942	1.00000					
	3	.16317	.55070	.81585	.94697	.99184	.99942	1.00000					
	4	.09324	.47203	.78788	.94697	.99184	.99942	1.00000					
	5	.05594	.42483	.78788	.94697	.99184	.99942	1.00000					
	6	.03730	.42483	.78788	.94697	.99184	.99942	1.00000					

Table II (continued)

m	r \ c	1	2	3	4	5	6	7	8	9	10	11	12
8	2	.28718	.64615	.84848	.94639	.98555	.99751	.99984	1.00000				
	3	.15664	.52867	.78757	.92463	.98135	.99751	.99984	1.00000				
	4	.08702	.44056	.74281	.91298	.98135	.99751	.99984	1.00000				
	5	.04973	.37762	.71733	.91298	.98135	.99751	.99984	1.00000				
	6	.02828	.33986	.71733	.91298	.98135	.99751	.99984	1.00000				
	7	.01989	.33986	.71733	.91298	.98135	.99751	.99984	1.00000				
9	2	.28235	.63529	.83620	.93665	.97984	.99515	.99926	.99996	1.00000			
	3	.15204	.51312	.76709	.90703	.97096	.99371	.99926	.99996	1.00000			
	4	.08293	.41983	.71181	.88544	.96643	.99371	.99926	.99996	1.00000			
	5	.04607	.34986	.67133	.87413	.96643	.99371	.99926	.99996	1.00000			
	6	.02633	.29988	.64829	.87413	.96643	.99371	.99926	.99996	1.00000			
	7	.016[illegible]	.26989	.64829	.87413	.96643	.99371	.99926	.99996	1.00000			
	8	.01053	.26989	.64829	.87413	.96643	.99371	.99926	.99996	1.00000			

Table II (continued)

m	r	1	2	3	4	5	6	7	8	9	10	11	12
10	2	.27864	.62694	.82663	.92879	.97487	.99274	.99842	.99978	.99999	1.00000		
	3	.14861	.50155	.75161	.89307	.96172	.98933	.99794	.99978	.99999	1.00000		
	4	.08002	.40510	.68926	.86384	.95220	.98766	.99794	.99978	.99999	1.00000		
	5	.04365	.33144	.63955	.84300	.94755	.98766	.99794	.99978	.99999	1.00000		
	6	.02425	.27640	.60317	.83218	.94755	.98766	.99794	.99978	.99999	1.00000		
	7	.01386	.23674	.58248	.83218	.94755	.98766	.99794	.99978	.99999	1.00000		
	8	.00831	.21307	.58248	.83218	.94755	.98766	.99794	.99978	.99999	1.00000		
	9	.00554	.21307	.58248	.83218	.94755	.98766	.99794	.99978	.99999	1.00000		
15	2	.26820	.60345	.79923	.90517	.95840	.98318	.99380	.99795	.99941	.99985	.99997	1.00000
	3	.13946	.47069	.70945	.85270	.93142	.97094	.98894	.99629	.99894	.99975	.99995	.99999
	4	.07276	.36837	.63148	.80397	.90509	.95865	.98406	.99467	.99853	.99968	.99995	.99999
	5	.03811	.28943	.56419	.76009	.88093	.94744	.97979	.99343	.99826	.99965	.99995	.99999
	6	.02006	.22850	.506[illegible]7	.72137	.85979	.93805	.97655	.99264	.99816	.99965	.99995	.99999
	7	.01062	.18145	.45702	.68795	.84224	.93096	.97455	.99234	.99816	.99965	.99995	.99999
	8	.00566	.14516	.415[illegible]5	.66003	.82875	.92646	.97375	.99234	.99816	.99965	.99995	.99999
	9	.00305	.11725	.38087	.63798	.81978	.92454	.97375	.99234	.99816	.99965	.99995	.99999
	10	.00166	.09593	.35340	.62247	.81558	.92454	.97375	.99234	.99816	.99965	.99995	.99999

Table II (continued)

	c	1	2	3	4	5	6	7	8	9	10	11	12
20	2	.26334	.59252	.78631	.89348	.94956	.97735	.99038	.99616	.99856	.99950	.99984	.99995
	3	.13543	.45708	.69050	.83363	.91568	.95978	.98201	.99248	.99709	.99896	.99966	.99990
	4	.06977	.35320	.60709	.77715	.88141	.94093	.97254	.98820	.99533	.99832	.99946	.99985
	5	.03601	.27345	.53471	.72513	.84835	.92205	.96287	.98372	.99349	.99766	.99925	.99979
	6	.01863	.21216	.47193	.67771	.81727	.90393	.95344	.97938	.99161	.99706	.99908	.99976
	7	.00966	.16501	.41753	.63473	.78857	.88706	.94472	.97546	.99024	.99658	.99897	.99974
	8	.00502	.12871	.37041	.59598	.76243	.87180	.93701	.97215	.98907	.99626	.99890	.99973
	9	.00262	.10073	.32968	.56125	.73899	.85841	.93054	.96958	.98828	.99609	.99888	.99973
	10	.00137	.07914	.29454	.53039	.71837	.84708	.92546	.96781	.98065	.99603	.99888	.99973
30	2	.25870	.58207	.77374	.88192	.94047	.97098	.98632	.99376	.99725	.99883	.99952	.99981
	3	.13170	.44449	.67279	.81536	.89995	.94787	.97386	.98739	.99415	.99739	.99889	.99954
	4	.06709	.33966	.58509	.75233	.85844	.92244	.95925	.97947	.99009	.99542	.99798	.99915
	5	.03420	.25974	.50914	.69395	.81775	.89618	.94341	.97050	.98530	.99301	.99684	.99864
	6	.01745	.19878	.44338	.64035	.77873	.86996	.92701	.96089	.98003	.99030	.99552	.99805
	7	.00891	.15226	.38644	.59128	.74172	.84434	.91053	.95101	.97448	.98739	.99410	.99740
	8	.00455	.11673	.33711	.54641	.70685	.81962	.89430	.94110	.96885	.98440	.99263	.99673
	9	.00233	.08958	.29438	.50543	.67412	.79599	.87856	.93138	.96328	.98144	.99118	.99607
	10	.00119	.06883	.25734	.46801	.64349	.76899	.86348	.90360	.95789	.97859	.98979	.99546

Table II (continued)

m	x	1	2	3	4	5	6	7	8	9	10	11	12
10	2	.25645	.57702	.76765	.87621	.93586	.96763	.98407	.99235	.99641	.99836	.99927	.99968
	3	.12994	.43853	.66436	.80654	.89214	.94172	.96943	.98442	.99228	.99628	.99826	.99921
	4	.06586	.33341	.57487	.74061	.84730	.91310	.95212	.97444	.98677	.99336	.99677	.99848
	5	.03339	.25358	.49756	.67961	.80328	.88338	.93318	.96297	.98014	.98969	.99482	.99748
	6	.01694	.19294	.43082	.62363	.76095	.85356	.91337	.95049	.97265	.98540	.99247	.99625
	7	.00860	.14686	.37319	.57241	.72071	.82419	.89321	.93739	.96455	.98063	.98978	.99480
	8	.00436	.11184	.32341	.52560	.68267	.79564	.87308	.92396	.95606	.97551	.98685	.99320
	9	.00222	.08521	.28061	.48327	.64740	.76874	.85391	.91114	.94804	.97086	.98443	.99216
	10	.00113	.06496	.24324	.44375	.61307	.74161	.83380	.89688	.93852	.96468	.98051	.98964